History effects and pinning regimes in solid vortex matter

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We propose a phenomenological model that accounts for the history effects observed in ac susceptibility measurements in YBa$_2$Cu$_3$O$_7$ single crystals [Phys. Rev. Lett. 84, 4200 (2000) and Phys. Rev. Lett. 86, 504 (2001)]. Central to the model is the assumption that the penetrating ac magnetic field modifies the vortex lattice mobility, trapping different robust dynamical states in different regions of the sample. We discuss in detail on the response of the superconductor to an ac magnetic field when the vortex lattice mobility is not uniform inside the sample. We begin with an analytical description for a simple geometry (slab) and then we perform numerical calculations for a strip in a transverse magnetic field which include relaxation effects. In calculations, the vortex system is assumed to coexist in different pinning regimes. The vortex behavior in the regions where the induced current density $j$ has been always below a given threshold ($j_c$) is described by an elastic Campbell-like regime (or a critical state regime with high critical current density, $j_c^0$). When the VS is shaken by symmetrical (e.g., sinusoidal) ac fields, the critical current density is modified to $j_c = j_{c}^<$ or $j_{c}^>$ at regions where vortices have been forced to oscillate by a current density larger than $j_{c}^>$. Experimentally, an initial state with high critical current density ($j_{c}^<$) can be obtained by zero-field cooling, field cooling with no applied ac field, or shaking the vortex lattice with an asymmetrical (e.g., sawtooth) field. We compare our calculations with experimental ac susceptibility results in YBa$_2$Cu$_3$O$_7$ single crystals.

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I. INTRODUCTION

The competition between vortex-vortex interactions and randomness determines the electromagnetic response of type-II superconductors in the mixed state.\(^1\) The interaction between vortices favors an ordered vortex lattice (VL) that opposes the disorder arising from random pinning centers and temperature. The combined effect of these interactions leads to a great variety of collective behaviors, making the VL a paradigmatic system to study an elastic object in a disordered environment.\(^2\)

The rich static phases of vortex matter in high-$T_c$ superconductors have drawn a great deal of attention in the past,\(^1,3,4\) but recently, much effort has been directed towards understanding vortex lattice dynamics both in low- and high-$T_c$ materials. Though details are not completely understood, consensus has been reached that the nucleation and annihilation of defects (e.g., dislocations) in the VL might play a major role in its response to an applied force.\(^5-16\) Reproducible states with distinct mobilities have been observed, and it has been argued that this reflects distinct degrees of topological order of the VL: a defective VL would interact more efficiently with pinning centers than an ordered one because of a reduction in its effective shear modulus. In particular, several experiments have revealed that, in certain conditions, an ac magnetic field or an ac current can assist the VL in ordering.\(^9-15,17\)

At first, two basic mechanisms have been invoked to account for the dynamic ordering of the VL in low-$T_c$ superconductors: a dynamic transition occurring at current densities above a certain threshold\(^18,19\) and an equilibration process where an ac magnetic field (or current) perturbation assists the VL in going from a supercooled disordered state into the more ordered equilibrium state. However, data have been reported that cannot be accounted for within either of these two frameworks. We refer to the recent findings in the vortex lattice in twinned YBa$_2$Cu$_3$O$_7$ (YBCO) single crystals.\(^14,15\) It was found that when vortices are shaken by a temporarily symmetric ac magnetic field (e.g., sinusoidal, square, triangular, etc.), they are driven into a mobile state, but if the shaking field is temporarily asymmetric (e.g., sawtooth), the lattice is trapped into a more pinned structure.\(^15\) As suggested in Ref. 15, the possibility of switching to a high-mobility VL state by applying a symmetrical ac magnetic field or to a low-mobility VL state by applying a temporarily asymmetric magnetic field wave-form is inconsistent with a naive equilibration process. In addition, the induced current density that participates in ordering or disordering the VL can be tuned to be comparable by adjusting the magnetic field amplitude and frequency. Disorder observed for higher field sweep rates than those that are used in symmetric oscillating fields that order the vortex lattice is a clear indication that the higher mobility of an ordered VL is not attained through a dynamical transition at high current densities.\(^15\) If this were the case, the VL should also order with the asymmetric ac field that is expected to induce a higher current density.\(^20\) This demonstrates, at least for YBCO, that the high or low VL mobility results from a collective dynamical process of a different kind that we believe is intrinsic to the oscillatory motion.

The attained degree of mobility is found to persist after interrupting the ac excitation for long periods of time (exceeding 1 h). As the ac field penetration is dependent on its amplitude $H_0$ and temperature $T$, different regions of the sample may remain in different pinning regimes depending on the detailed thermomagnetic history of the sample.\(^14\) The dynamical rearrangement of the VL occurs in the outer zone of the sample where the penetration of the ac magnetic field
induces ac currents that force vortices to move. An increment of \(H_0\) or \(T\) will result in an increase in the penetration depth of the ac field. As the ac field further penetrates into the sample, the VL mobility changes. In Ref. 14, we argued that, as long as the current density does not exceed the critical one (so that a small fraction of the vortices moves out of their pinning sites), the VL will remain in the low-mobility state. In a bulk-pinning-dominated regime, this will happen in the inner nonpenetrated portion of the sample. As \(T\) or \(H_0\) increases this region will shrink and, eventually, disappear. The robustness of the states involved and their spatial distribution in the sample are the main ingredients leading to the observed memory effects in Ref. 14.

In this paper, we present analytical and numerical calculations for the field and current distributions, as well as for the ac susceptibility \(\chi_{ac}\) in samples with inhomogeneous pinning. If history effects and flux creep can be neglected, the current and flux densities inside the superconductor are often well described by the Bean critical-state (CS) model which assumes a critical current density that is magnetic field independent and a null lower critical field \(H_{c1}\) \((H_{c1} = 0, B = \mu_0 H)\). Exact analytical solutions have been obtained for the perpendicular geometry which is more appropriate for our experimental arrangement. For more general situations, as in our particular case, a time-dependent theory is required. If it is assumed that the critical current density does not vary over the sample thickness, this dynamic theory can be formulated in terms of a one-dimensional integral equation as discussed by Brandt. With the use of an appropriate constitutive relation \(E(j)\) it is possible to compute the flux motion (and the magnetization) in strips, disks, etc., with homogeneous or inhomogeneous \(j_c\) when the applied field is cycled. From the magnetization curves, we calculate the susceptibility. These results are compared with measured ac susceptibility data obtained in a twinned YBCO crystal following a specific protocol to control the spatial pattern of the dynamical behavior of the VL in the sample.

As an example, the VL can be prepared by zero-ac-field cooling (ZFacC), cooling the sample below the critical temperature before applying the magnetic ac field. This traps the VL in a low-mobility state with a high critical current density \(\sim j_c^x\). If a sinusoidal ac field is then applied, the VL will become more mobile in the penetrated outer zone of the sample and it will be characterized by a lower critical current density \(\sim j_c^u\). Vortices in the inner region of the sample, where the induced current density \(j\) has been always below \(j_c^u\), will remain in a more pinned structure in which vortices oscillate confined in their pinning potential wells.

The critical current patterning so defined may be observed, for example, by removing the ac field and applying an increasing ac probe to measure the ac susceptibility \(\chi_{ac}\). This is the basic protocol that we have developed in calculations and in our ac susceptibility measurements to show evidence of the different local degrees of VL mobility. Analytical calculations that qualitatively introduce our model are presented in Sec. II A. In Sec. II B we present our numerical calculations of \(\chi_{ac}\) which take into account a more adequate transverse geometry and also include the effect of flux creep.

In calculations, we consider two possible values for the critical current density, though it should be kept in mind that it is observed experimentally that \(j_c\) depends on the number of cycles of the “shaking” ac field (see Ref. 15). Experimental results are described in Sec. III and conclusions are drawn in Sec. IV.

II. THEORY

For symmetrical shaking fields, we can argue that the VL in the inner portion of the sample will remain in a low-mobility state as long as the inner current density does not exceed the critical current density. In this case, a Campbell-type regime may describe the elastic pinning of vortices that are confined to oscillate inside a potential well.

The main criterion to discriminate between a Campbell and a critical-state regime arises from comparing the scale of the vortex displacement \(\Delta x\) produced by the induced currents and the size of the pinning potential well \(r_p\). The application of an ac magnetic field results in a periodic compression of the VL, with a maximum effective vortex displacement at the sample’s boundary. This displacement can be easily estimated from the variation of the lattice parameter

\[
a_0 = \sqrt{\frac{2}{\phi_0}} \frac{\phi_0}{\sqrt{3} B_{dc}},
\]

which occurs after applying a perturbation in the dc field \(B_{dc}\). For a sample with no pinning and with \(\mu_0 H = B\) (i.e., neglecting \(H_{c1}\)), the vortex displacement at the sample perimeter arising from an ac field of amplitude \(B_0(B_0 \ll B_{dc})\) is

\[
\Delta x \approx \frac{1}{2} \frac{B_0}{B_{dc}} R,
\]

where \(R\) is the sample radius. If \(\Delta x\) is smaller than \(r_p\), the elastic pinning regime usually describes satisfactorily the penetration of magnetic flux. As the ac field amplitude increases, the region in the sample where the elastic regime applies naturally shrinks. We shall come back to this criterion further on, and momentarily we shall address the problem of a superconductor with nonuniform critical current density.

A. Statics: Analytical calculations

We start by calculating the ac susceptibility of an infinite slab \(|x| \ll A\) in a longitudinal magnetic field for which the inner critical current density \(j_c^{int} = j_c^x\) \((|x| \ll \lambda_{c} \ll A)\) is larger than the outer one \(j_c^{ext} = j_c^u\) \((x, |x| < A)\) [see Fig. 1(a)]. This first calculation will provide a qualitative description of the observed ac field amplitude dependence of the ac susceptibility. We shall not consider at present that the ac field modulates the local critical current density, and we will suppose a given fixed spatial distribution of \(j_c\). For illustrative purposes we also take \(j_c \rightarrow \infty\) so that the outer region of the sample \(x_d < |x| \ll A\) is in the CS whereas there is perfect shielding (Campbell penetration length \(\lambda_{c} \ll x_d\) for \(|x| < x_d\). The calculation of \(\chi_{ac}\) in this simplified situation is
describe the case of a fixed boundary separating high- and
where $x < x_d$ (dark gray) and $x > x_d$ (light gray). (b) Field profiles inside the sample for $j_c \rightarrow \infty$.

straightforward and follows from known results for the critical state in a slab\(^{33}\) if one notes that (i) for ac field amplitudes below $H_d = j_c^c (A - x_d)$, $X_{ac}$ is the well-known CS susceptibility for an infinite slab with $j_c = j_c^c$; (ii) for fields higher than $H_d$, the sample magnetization is given by the sum of the magnetization due to the outer region of total width $2(A - x_d)$ in the CS and that due to the inner region of width $2x_d$ with perfectly diamagnetic susceptibility $\chi' = -i\chi'' = -1$ [see Fig. 1(b)]. For an applied ac magnetic field $H_{ac} = H_0 \cos(\omega t)$, the susceptibility is, for $H_0 = H_d$,

$$\chi' = -1 + \frac{1}{2} x,$$

$$\chi'' = \frac{2}{3\pi} x,$$

and, for $H_0 > H_d$,

$$\chi' = \frac{1}{\pi} \left[ \frac{1}{2} x^* - 1 \right] \cos^{-1} \left( 1 - \frac{2}{x^*} \right) + \left( -1 + \frac{4}{3x^*} - \frac{4}{3x^*^2} \right) \times (x^* - 1)^{1/2},$$

$$\chi'' = \frac{1}{3\pi} \left( \frac{6}{x^* - 1} + 4 \right) \left( \frac{A - x_d}{A} \right),$$

where $x = H_0 / H_p$, $H_p = j_c^c A$, and $x^* = H_0 / H_d = (H_p / H_d) x = [A/(A - x_d)] x$.

As an example, we plot in Fig. 2(a) the calculated imaginary component $\chi''$ as a function of the normalized ac magnetic field $H_0 / H_p$ for different values of $x_d/A$. These results describe the case of a fixed boundary separating high- and low-mobility regions of the VL (in each curve the boundary is at a different position). We will address now the more realistic situation where the ac field modifies the position of the boundary.

As was discussed above, the elastic pinning regime is valid for ac fields of low amplitude, where vortex displacements at the sample boundary do not exceed the size of the pinning potential well. As the ac field amplitude is increased, the region in the sample where this condition is valid will be gradually reduced. As a consequence, the curves plotted in Fig. 2(a) for fixed $x_d$ can not be valid for an arbitrary high ac field amplitude. A more realistic approach has to consider that $x_d$ will gradually reduce for sufficiently strong fields. This reduction should be monotonic in the ac field amplitude.

It is useful to define a “minimal penetration susceptibility” that represents the ac susceptibility of a sample that initially contains a homogeneous low-mobility VL and is subjected to an increasing ac magnetic field starting from zero. These curves are represented in Fig. 2(a) by dashed lines and will be discussed in the following paragraphs.

The application (in the elastic regime) of a small perturbation field $b_{ac}$ (ac or dc) induces currents that exert a pressure on the vortices at the sample boundary that propagates inside the sample due to the VL elasticity. In a wide slab, the perturbation decays exponentially, with a characteristic length $\lambda_C$, the Campbell penetration depth.\(^{30–32,35}\)

We suppose first an increasing ac field amplitude starting from $H_0 = 0$. As $H_0$ is increased above a given threshold, $h_T$, vortices in the outer part of the sample are driven out of the Campbell state as they are forced to move out of their pinning sites. The subsequent oscillatory motion of the vortices produces a reordering of the VL that results in a local reduction of $j_c$ to $j_c^c$. Here, $h_T$ is related to the depth of the pinning potential well in which vortices oscillate in the Campbell regime.

We can estimate $x_d$ as a function of $H_0$ for a semi-infinite sample ($x = 0$). A perturbation $b_{ac}$ penetrates the sample as $\Delta B(x) = b_{ac} \exp(-x/\lambda_{ac})$, where the ac penetration depth is given by $\lambda_{ac}^2 = \lambda^2 + \lambda_C^2$ with $\lambda$ the London penetration depth.\(^{35,36}\) In the case we are interested in $\lambda_C = (c_{11} / \alpha_L)^{1/2}$, where $c_{11}$ is the compressibility of the VL and $\alpha_L$ is the Labusch parameter (in general for weak pinning $\lambda_C \gg \lambda$ and $\lambda_{ac} \sim \lambda_C$).
To determine whether the elastic limit is applicable or not, we estimate the vortex displacement at the sample surface as

$$\Delta x \sim \int_0^{\infty} \frac{\Delta B(x)}{B_{dc}} dx.$$  \hspace{1cm} (5)

As discussed above, for displacements $\Delta x \sim r_p$, or larger, the elastic regime is not expected to be applicable. We find from Eq. (5) that this occurs for an applied perturbation $b_f = b_T \sim r_p N \alpha_l \mu_0$, where we have used $\lambda_{ac} \sim \lambda_c$ and $c_1 \sim B_{dc} / \mu_0 (\mu_0 = \mu_0 h_T)$. If the VL is not strongly perturbed, then $\alpha_l b_T = j_c B_{dc}$. In the semi-infinite sample, it is evident that this relation establishes the applicability of the elastic Campbell regime at any point inside the sample. Then, we can assume that, if the VL is in a low-mobility state ($j_c = j_c^>$) in the whole sample and the ac field amplitude is increased from zero, $x_d$ will reduce monotonically from $x_d = A$ to a value given by $\Delta B(x_d) = b_T \sim \sqrt{j_c}$. In this picture, the ac susceptibility of a slab is given by Eqs. (1)–(4) with $x_d$ not a constant anymore but depending on the applied ac field amplitude:

$$H_0 \ll h_T: \quad x_d = A,$$

$$H_0 > h_T: \quad x_d = A - \frac{H_0 - h_T}{j_c}.$$  \hspace{1cm} (6)

Figure 2(a) shows in dashed lines the “minimal penetration curve” as defined above for the imaginary component $\chi''$ as a function of the normalized ac magnetic field $H_0 / h_T$ for $h_T = 0.2, 0.4$, and $0.6 h_p$. In Fig. 2(b), we represent the ac susceptibility of a slab predicted by this simplified picture where the internal region contributes with a higher screening capability and the dissipation decreases. However, as $\Delta B(x_d) < b_T$, $x_d$ remains unchanged and equal to its initial value, $x_d^0$. For $H_0 > h_T$ the boundary between low and high VL mobility progressively penetrates into the slab and $x_{ac}$ follows the minimal penetration curve. Finally, for $H_0 = h_T$ the sample is fully penetrated and the CS regime is recovered. The convenience of this calculation will become clear when we describe in Sec. IV the protocols followed in experiments.

**B. Numerical calculations**

**1. Computational method**

The original Bean critical-state model has been extended to different geometries, including long strips$^{24,26}$ and circular disks$^{23,25}$ in a perpendicular magnetic field. These geometries are more appropriate for our experimental situation. The results obtained analytically can be reproduced by an algorithm developed by Brandt$^{27,28}$ and reviewed in Ref. 28.

The magnetic field generated by these currents is given by Ampère’s law. For the strip, the total field perpendicular to its surface evaluated at $x = 0$ has the form

$$H_s(0,x) = H(y) = \frac{1}{\pi} \int_{-A}^{A} \frac{J(u)}{y - u} du + H_a.$$  \hspace{1cm} (7)

As the current flowing in the strip is induced by the external magnetic field, the symmetry $J(y) = -J(-y)$ can be used in Eq. (6). Considering the electric field $E(y,t) = \delta(y,t)/L$, induced by the time-dependent magnetic flux $\phi(t) = \mu_0 \int_0^t H(u) du$, one gets an integro-differential equation for the sheet current. This equation has been discretized$^{27,28}$:

$$J_i(t) = \sum_{j} K_{ij} \left[ J_j(t) - 2 \pi u_j H_a(t) \right],$$  \hspace{1cm} (8)

where $K_{ij} = \int_0^L \frac{w_j}{w_i} \operatorname{ln} \left[ \frac{u_i - u_j}{u_i + u_j} \right] u_i < u_j \neq 0$, and $w_i = u_i^{*} / (4 \pi \ln u_i)$ is the weight function.

Having obtained the sheet current as a function of time by solving Eqs. (7) and (8), we evaluate the magnetic moment $\mathbf{m}$ of the sample that points in the $\hat{x}$ direction as

$$\mathbf{m} = \frac{1}{2} \int_0^A H(y) dy.$$  \hspace{1cm} (9)

which for the strip results in

$$\mathbf{m} = -\hat{x} M L, \quad M = \int_{-A}^A y J(y) dy.$$  \hspace{1cm} (10)
If the applied field is sinusoidal, $H_0(t) = H_{ac}(t) = H_0 \cos(\omega t)$ where $\omega = 2\pi / T$, it is possible to define the ac susceptibility coefficients as

$$\chi_n = \frac{\omega}{\pi H_0} \int_0^T M(t) \cos(n \omega t) \, dt.$$  

(11)

$$\chi_n' = \frac{\omega}{\pi H_0} \int_0^T M(t) \sin(n \omega t) \, dt.$$  

(12)

To obtain the susceptibility we solved Eqs. (7) and (8) by a fourth-order Runge-Kutta method with automatic control of the step size.38 We chose the constitutive relation suggested above:

$$E(J) = E_c \left( \frac{J}{J_c} \right)^p \frac{\rho}{J}.$$  

(13)

with $J = |J|$.  

In the Bean critical-state model $E(J)$ is strongly nonlinear and has an abrupt increment at $J = J_c$. For the constitutive relation of Eq. (13) this occurs for $p \rightarrow \infty$. For $1 < p < \infty$ it leads to a nonlinear relaxation (approximately logarithmic) of $M(t)$ caused by the nonlinear magnetic flux diffusion. As $p$ decreases the relaxation becomes stronger and in the limit $p = 1$ the behavior is ohmic, with linear magnetic flux diffusion and exponential relaxation of $M(t)$. In our simulations we chose $p = 20$, as $p = 10$ would blur the flux front due to flux creep and $p = 50$ would require long simulation times to detect relaxation effects. Note that, in general, only the combination $E_c / J_c^p$ is relevant [Eq. (13)], and the voltage criterion, $E_c$, and $J_c$ are not independent. While in the critical state limit $E_c$ is irrelevant, in the ohmic regime $E_c / J_c = \rho$ is the resistivity. Within the dynamical approach that we describe, $J_c$ has a different meaning. As the current is time dependent (it may depend on the rate of variation of the magnetic field), $J_c$ is related to the degree of vortex mobility, where low values of $J_c$ represent highly mobile vortex structures.

The space variable was discretized with $N = 50$ nonequidistant points (results with $N = 30$ showed no significant difference). The tabulation was performed so that the weight function $w(x) = u'(x)$ vanished at the integration boundary to remove the infinity in the integrand occurring at $u \rightarrow 1$ at $t = 0$ (see Ref. 27):

$$u(x) = \frac{3}{2} x - \frac{1}{2} x^3,$$

$$w(x) = \frac{3}{2} (1 - x^2).$$

For the ac field $H_{ac}(t) = H_0 \cos(\omega t)$ measured in units of $J_{c1} d$, $\omega = 2 \pi / (\mu_0 J_c d A)$ which, in units of $A = 2 E_c / (\mu_0 J_c d A)$, is $\omega = 1 / \mu_0$. The ac susceptibility was calculated after two complete ac cycles, which are necessary to reach a stationary state in the calculation of the hysteresis loop $M[H(t)]$.

2. Current and field profiles: ac susceptibility

In our calculations, the critical sheet current $J_c$ at each point $y_i$ inside the strip depends on the magnetic history of the sample. As was discussed above, we assume that $J_c$ can take two different values, $J_c^< \,$ or $J_c^>$ (now both finite) depending on the sheet current $J(y_i)$ that flowed in $y_i$ in the previous iteration. For a given $H_0$, if $J(y_i) > J_c^>$, then $J_c(y_i)$ is replaced by $J_c^<$ (which has been arbitrarily chosen as unity) and the calculation is restarted. Clearly, the magnetic history of the sample fixes the initial distribution of $J_c(y_i)$ in Figs. 3 and 4 we show the result of our calculations starting with zero ac field and different initial conditions. In Fig. 3 we show the field (upper panel) and the current (lower panel) distributions for a strip with the following initial critical sheet current distributions: $J_c = J_c^<$ (left), $J_c = J_c^>$ (center), and $J_c = J_c^<$ for $|y/A| > 0.75$ and $J_c = J_c^>$ for $|y/A| \leq 0.75$ (right panels).

In the protocol for the calculation of ac susceptibility (Fig. 4) we sweep the ac field from zero. The minimal penetration curve is calculated as follows. We begin with $J_c = J_c^<$ for the whole strip (low mobility initial state, center panels in Fig. 3). The field is increased from zero and we compute the field and current profiles. If the sheet current exceeds $J_c^>$ at any point of the strip, then $J_c$ is changed to $J_c^<$ at that point and

![Figure 3](https://example.com/figure3.png)

**FIG. 3.** Field profiles (top) and current profiles (bottom) obtained numerically for a field $H_0 \cos(\omega t)$ increasing from zero and evaluated at $H_0$, for different $H_0 > 1$. Left: homogeneous sample, $J_c^* = J_c^* = J_c$. Center: penetration for the minimal penetration curve $J_c^* = 2 J_c^>$. Right: profiles obtained increasing $H_0$ with initial $x_d = 0.75 A$. Calculations are for a thin superconducting strip. See the text.
the calculations are restarted. This generates by construction the smallest outer region with \( J_c = J_c^0 \) for each given ac amplitude \( H_0 \) (so that \( J < J_c^0 \) in the whole sample). The field amplitude \( H_0 \) is increased again and the procedure repeated. Note that the region with \( J_c = J_c^0 \) shrinks towards the center of the strip as \( H_0 \) is increased. In the calculation of \( \chi_{ac} \) the field has to be varied in a complete cycle, but it is clear that the critical current density distribution is fixed after the first quarter of cycle.

Now we calculate the rest of the curves that result for a strip with an initial distribution of \( J_c \) as in the right panels of Fig. 3. To obtain the initial state, we apply a magnetic field of a given amplitude and we define the initial distribution of \( J_c \) following the procedure described above. We then calculate the ac susceptibility increasing \( H_0 \) from zero and check for the condition to replace \( J_c^0 \) by \( J_c^\ast \). This occurs for field amplitudes larger than the one that defined the initial state. Note that from this field amplitude on, the calculated susceptibility coincides with the minimal penetration curve.

It is worth noting that in a strictly two-dimensional (2D) model, it is not possible to have an abrupt change in \( J_c \), because this causes a local divergence in \( J \) and vortices would always move. To solve this inconvenience, in our calculations, the critical current density changes smoothly from \( J_c^0 \) to \( J_c^\ast \) over distances of the order of \( d \) [where the 2D treatment breaks down\(^25\)].

In Fig. 4 (left panels) we took \( J_c^0 = 2J_c^\ast \). In Fig. 4 (right panels) we compare the minimal penetration curves for different values of \( J_c^\ast \) (1, 1.5, and 2). As we shall see in the next section, the protocol followed in the numerical calculations is particularly convenient to directly compare the numerical results with the experimental ones. In the experiments that are described below, we are able to prepare, in a highly reproducible way, an initial state with low mobility in the whole sample (that is, described in calculations with \( J_c^\ast \)) or an initial state with high mobility at the outer region of the sample (a situation that corresponds to \( J_c^\ast \) in the outer region and \( J_c^\ast \) in the inner region).

### III. Experiment

We measured the ac susceptibility of a twinned YBCO single crystal\(^{39}\) using a standard mutual inductance technique. The characteristics of the crystal are the following: dimensions \( 0.56 \times 0.60 \times 0.03 \) mm\(^3\), \( T_c = 92 \) K, and \( \delta T_c = 0.3 \) K determined by ac susceptibility \((h_{ac} = 1 \) Oe\) at zero dc field. In our measurements the ac field is applied parallel to the \( c \) axis of the sample. The dc magnetic field \( H_{dc} = 3 \) kOe was applied at 20° out of the twin boundaries to avoid the Bose-phase glass. Further details can be found in Ref. 14.

The generally unknown dependence of the critical current on temperature makes \( \chi_{ac} \) measurements versus ac field amplitude at constant temperature the best choice for comparison with numerical results. The experimental procedure, based on our previous results, is as follows.

(i) We set the VL in a low-mobility state by applying 10\(^4\) cycles of an ac magnetic field with a sawtooth wave form (10 kHz, 8 Oe; see Ref. 15). This field completely penetrates the sample.

(ii) We define the initial state by applying 10\(^5\) cycles of a sinusoidal ac field of a given amplitude at 10 kHz.

(iii) This ac field is turned off and we measure ac susceptibility increasing the amplitude of the ac field probe from zero, with a fixed measuring frequency of 10 kHz.

Results are shown in Fig. 5 for a fixed temperature and \( H_{dc} = 3 \) kOe. Different initial states were prepared applying 10\(^5\) cycles of a sinusoidal ac field of amplitudes 0, 1.6, 2.4, 3.2, 4, 4.8, and 8 Oe.

The similarities with numerical simulations for \( J_c^\ast = 2J_c^\ast \) shown in Fig. 4 are evident. The marked change in the response of the ac field amplitude that defines the initial state supports the scenario of vortices being shaken or not out of the pinning centers on either side of a well-defined boundary. The effective critical current density of the more mobile state can be estimated from the position of the dissipation peak. From Fig. 4 we know \( j_c^\ast \approx H_0^{peak}(0.75d) \). With \( d = 30 \) µm and \( H_0^{peak} = 3.5 \) Oe, \( j_c^\ast \approx 1250 \) A/cm\(^2\), and thus \( j_c^\ast \approx 2500 \) A/cm\(^2\) (at 85.2 K, see Fig. 5).

We also observe that the inductive component does not approach \(-1 \) as \( H_0 \) is reduced to zero. This is probably a consequence of the finite penetration depth in the Campbell regime\(^{29,40}\). We can determine some of the relevant parameters in this regime, estimating from the saturation value \([\chi'(0) \approx -0.9]\), the value for \( \lambda_c \approx \lambda_{ac} \approx 10 \) µm \([\ll \lambda \approx 0.6 \) µm (Ref. 41)] and the size of the potential well \( r_p \approx 50 \) A [Eq. (5), \( r_p \approx \lambda_{ac}h_{T}/h_{dc} \) with \( h_T \approx 1.5 \) Oe and \( H_{dc} = 3 \) kOe]. The value for \( h_T \) was estimated as the minimum ac amplitude capable of modifying the measured susceptibility (starting from a low-mobility VL).

We note that from these estimates \( r_p \) turns out to be of the
order of the coherence length $\xi \approx 60$ Å at our working temperature \( [\xi \sim \xi_0 (1 - T/T_c)^{-1/2}] \) with $\xi_0 \approx 20$ Å (Ref. 41]. The Labusch parameter is

$$\alpha_L \sim \frac{j_c B_{dc}}{r_p} = \frac{j_c \phi_0}{a_0^2} \sim 10 \text{ N/m}^2 \text{ cm}^2 \text{ s}^{-1} \left( \frac{100 \text{ dyn/cm}^2}{a_0^2} \right).$$

**A. Frequency dependence of the penetration depth**

Within the dynamical model described above, an increase in the frequency of the ac field induces a higher ac current (on the average) with a smaller penetration depth. One can gain some insight by considering a stationary state where a constant field ramp $H_a$ is applied. In the stationary state the current induced by the field ramp does not change with time and the electric field is obtained as $\nabla \times E = -\mathbf{B}_d$, where $\mathbf{B}_d = \mu_0 H_a$, implying $E \sim B_d$. Inverting Eq. (13) we get $J \sim (B_d)^{1/p}$, which shows how $J$ grows as $B_d$ is increased. Taking $J = \text{constant}$ in, for example, Eq. (7), we arrive at the same result.

The reduction in penetration due to an increment in the frequency can be deduced using an interesting scaling property of Eq. (7) (Ref. 42]. Explicitly, if the unit of time is changed by a factor $k$ and time is expressed as $\tilde{t} = t/k$, then the new functions $\tilde{J}(\tilde{t}) = J(t) k^{1/p-1}$ and $\tilde{B}_d(\tilde{t}) = B_d(t) k^{1/p-1}$ satisfy the same equations. The resulting magnetic field scales with the same factor. If we consider a periodic magnetic field $H_{ac}(t) = H_0 \sin \omega t$, this scaling property implies that if the frequency is increased by a factor 10 and the ac field amplitude is also increased but by a factor $10^{(1/p-1)}$, the field penetration depth inside the sample will remain unchanged.\(^{42}\) This leads to the immediate conclusion that if the frequency is increased but the amplitude of the field is maintained constant, then the penetration depth will be smaller.

This behavior is observed experimentally. In Fig. 6 we compare the imaginary part of the susceptibility measured at 10 kHz after applying $10^5$ cycles of a sinusoidal ac field of a given amplitude at 100 kHz [Fig. 6(a)] and at 10 kHz [Fig. 6(b)]. As the amplitude of the measuring field increases, the susceptibility senses the VL mobility in the regions closer to the center of the sample. As the change in the response towards the low-dissipation branch is first observed in Fig. 6(a), it is clear that the penetration at 100 kHz is smaller than at 10 kHz. The reason for this is that, in the former case, the high-mobility outer region is necessarily smaller as implied...
by the susceptibility measurements (note that the susceptibility is measured at 10 kHz). From this difference and applying the scaling law discussed above, we can estimate the exponent $p$ in Eq. (13). As an example, the penetration depth of 3.2 Oe at 100 kHz is approximately equal to the penetration of 2.7 Oe at 10 kHz. Assuming a linear dependence of the penetration depth with the ac field amplitude, we obtain $p \sim 15$, which is close to the value we used in our simulations.

B. Temporarily symmetric and asymmetric ac magnetic fields

We have suggested in Ref. 15 that a change in the mobility of the VL may be related to a change in its degree of order and that these changes are intrinsic to the oscillatory dynamics. Molecular dynamics calculations support the scenario where defects are healed by a temporarily symmetric oscillation and created by a temporarily asymmetric oscillation. These simulations also show that the mean velocity of the VL increases as the number of defects decreases.

In the previous section we showed that it is possible to increase vortex mobility (or to order the VL) by applying a temporarily symmetric ac field, and we proved that this reordering occurs in the outer penetrated region of the sample, i.e., in the regions where induced currents have densities above the critical current density. We also know that a temporarily asymmetric field produces the opposite effect; it leads the VL into a low-mobility (disordered) state. Then, the following questions arise: (i) Does the penetration depth of the disordered region have a similar dependence on ac amplitude as the penetration depth of the ordered region for symmetrical ac magnetic fields? (ii) Does the amplitude of the asymmetric field have to be above a certain threshold to produce any effect?

Related to this last question we found that, for the application of a symmetrical field, the minimum amplitude that produces some reordering is $\sim 1.5$ Oe, and this value may be related to the depinning of vortices out of their pinning sites. As we shall see, however, the threshold amplitude for the asymmetric field to produce some disordering is twice as high; thus the process that prevents the VL from switching to a more pinned state should have a different origin.

The experimental protocol for the asymmetric ac fields is very similar to the one described above for symmetric fields, but was modified so that the mobility of the initial state is low. The following procedure is used.

(i) We generate a high-mobility VL in the whole sample by exposing it to $10^4$ cycles of an oscillating magnetic field with a sawtooth wave form (8 Oe amplitude and 10 kHz) followed by $10^5$ cycles of a sinusoidal ac field of the same amplitude and frequency. This leads to a high-mobility VL in the whole sample.

(ii) We define the initial state by applying $10^5$ asymmetric ac magnetic field cycles (i.e., sawtooth) of a given amplitude, $H_0^i$ (10 kHz).

(iii) We measure ac susceptibility with a sinusoidal ac probe as a function of amplitude, starting from amplitude zero (measuring frequency 10 kHz).

Results are plotted in Fig. 7 for the same temperature and field conditions of Figs. 5 and 6. We investigated different initial states prepared by the application of a sawtooth wave form ac field with amplitude, $H_0^i$, between 0 and 8 Oe (we show the results for $H_0^i=0, 3.2, 4,$ and 8 Oe). The first observations are that there seems not to be a clear boundary separating two regions with different vortex mobilities as in Fig. 5 and that changes in susceptibility are important only for $H_0^i \sim 3$ Oe or higher. With asymmetric ac fields, the curves for the different $H_0^i$ resemble the minimal penetration curves of Fig. 4 for different $J_c^{ini}$'s. The fact that there is no clear boundary between two regions with associated different mobilities of the VL might be explained by the difference in the sweep rate of the sawtooth wave form that implies a different penetration depth in the ramp up and down of the field. The measuring ac field can also contribute to blur the boundary as it tends to counteract the effect of the sawtooth ac field. On the other hand, we cannot obviate the fact that the shape of the susceptibility does not favor its observation. Finally, another possible explanation for this behavior is related to the mechanism that leads to the decrease of the VL mobility. In Ref. 15, we proposed that this decrease is related to an increase in the density of topological defects due to plastic distortions in the VL that are produced by the temporarily asymmetric induced currents (cf. Fig. 6; it is implicitly shown that higher field rates induce higher currents). The observation of a minimum amplitude of $\sim 3$ Oe required to produce a strong effect in the mobility of a moving lattice supports this assumption. For plastic distortions to occur, we
expect that vortices should move around one lattice parameter. For a sample in the critical state, the displacement at the sample boundary caused by a field that penetrates a distance \( d \) is

\[
\Delta x = \frac{1}{4} \frac{H_0}{B_{dc}} d
\]

(for \( d \) smaller than the sample radius). For a sample of radius 250 \( \mu \text{m} \) in a 3 kOe dc field and 3 Oe ac field (6 Oe peak to peak, and 5 Oe of full penetration) the total vortex displacement at the sample periphery is \( \sim 0.1 \mu\text{m} \), which is of the order of the lattice parameter \( a_0 = 0.09 \mu\text{m} \).

We note that although an important portion of the VL is moving by the application of 3 Oe ac field, only the portion near the sample boundary is initially distorted. This is not what occurs during VL ordering, where all the vortices that move by the action of a symmetrical ac field organize in a more ordered structure. It is possible that the defects generated at the outer region of the sample by the sawtooth wave form diffuse deeper into the sample assisted by the moving lattice. As disorder is produced only for high ac fields (\( H_0 > 3 \) Oe) that penetrate an important fraction of the sample, a significant portion of the VL will be contaminated after a certain number of oscillations. This description is supported by the direct observation in molecular dynamics simulations of the high mobility of dislocations in VL forced by oscillating forces.

**IV. CONCLUSIONS**

We have presented a phenomenological model that accounts for the history effects in the ac response of YBa\(_2\)Cu\(_3\)O\(_7\) single crystals. The model is based on the effect of inhomogeneous volume pinning in the ac susceptibility. We obtained analytical expressions for a simple geometry to address qualitatively the details of the model, and then we presented numerical results by solving Maxwell’s equations for a more realistic situation.

We presented new experimental results that are adequate for comparison with the numerical calculations. The very good qualitative agreement between them indicates that we have captured the essence of what occurs in relation to the different dynamical responses of the VL inside the sample.

The existence of a threshold sawtooth ac magnetic field amplitude required to lower the VL mobility supports the hypothesis of a ratchet tearing of the VL produced by temporarily asymmetric fields as was suggested in Ref. 15.

Some of the differences between experimental and numerical results are probably a consequence of an intrinsic property of the experimental technique that undoubtedly perturbs the state of the VL and in a way that may depend on the initial conditions. A sinusoidal ac field increases vortex mobility and the measurement itself can mask the effect of a previously applied asymmetric ac field. Besides, \( J_c \) is not restricted to take only two values. Future experimental and theoretical work is required to further understand the oscillatory VL dynamics.

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